Optimality of quasi myopic strategies for multi-stock discrete time market with exponential utility function

Jianguang Liu
Department of Mathematics, Trent University, Ontario, Canada

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Abstract

This paper presents a multi-stock discrete time market model. In this model, we consider an optimal solution in the multi-stock portfolio selection. Specially, the model allows that the optimal strategy to maximize an exponent function of the expected value is quasi myopic.

Key words: discrete time, multi-stock market, optimal myopic strategies, utility function.

1 Introduction

The paper addresses the question of optimality of myopic strategies for multi-stock discrete time market. We found that optimal portfolio strategy for maximizing the expected exponential utility at the end of a trading period in a financial market. Preferences of the agent in consideration are described by an increasing concave function $U(x) = -e^{-cx}$, trading dates occur at discrete time periods, with optimal portfolio problem that $EU(X_T)$ is to be maximized, where $X_T$ is the total value of time $T$.

Merton (1969) was the pioneer who studied the issue by understanding the behavior of an individual who acts as a market price-taker seeks to maximize the expected utility of terminal wealth in continuous time model. Many researchers have extended Merton’s dynamic asset allocation model over the past decades. And it is also important to extend the class to discrete time models that allow myopic and explicit optimal portfolio strategies. In the literature, Simth(1967), Leland(1968), Mossin(1968), Samuel-
son(1969), Francis(1976), Li and Ng(2000), manage to study the portfolio selection in discrete-time models.

Modern portfolio theory is based on the implicit assumption that risk tolerance is proportional to wealth, though empirical validation or invalidation of this assumption remains to be done. In the literature, similar utility function has been discussed for the motivation of exponential utility maximizing in Kramkov and Schachermayer(1999), Schachermayer(2001), Christian and Stricker (2002), Pedersen and Satchell(2003), Nan(2007).

In fact, the optimal strategies for continuous time market models are myopic if the risk free rate, the appreciation rate, and the volatility matrix are random processes that are supposed to be currently observable. The case of random coefficients was discussed in Karatzas and Shreve (1998) and Dokuchaev and Haussmann (2001). The solution leads to myopic strategies goes back to Merton(1969) in continuous time model. There are several special cases when investment problem allows explicit solution for discrete time, for some cases, optimal strategies are myopic (see Leland (1968), Mossin (1968), Hakansson (1971)). And in this paper, we found that our solution with exponential utility function is almost myopic.

The present paper continues study started in Dokuchaev(2007), where it was studied when an optimal investment strategy is myopic with utility function: $U(x) = \ln x$, $U(x) = \delta^{-1}x^\delta$. In fact, the basic restrictions for the class of models were similar to the ones that ensure optimality of myopic strategy in continuous setting. The novelty of the paper is the use of exponential function as the utility function. In addition, the setting of the discrete time model is modified, and we will implement some simple numerical experiments to verify our myopic strategies.