Portfolio Execution Cost Problem in the Presence of Market Impact

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Abstract

When liquidating a portfolio of large blocks of risky assets, an institutional investor wants to minimize the cost as well as the risk of execution. An optimal execution strategy minimizes a weighted combination of the expected value and the variance of the execution cost, where the weight is given by a nonnegative risk aversion parameter. The execution cost is determined from price impact functions. In particular, a linear price impact model is defined by the temporary impact matrix \( H \) and the permanent impact matrix \( \Gamma \), which represent the expected price depression caused by trading assets at a unit rate. In this paper, we analyze the sensitivity of the optimal execution strategy to estimation errors in the impact matrices under a linear price impact model. We show that, instead of depending on \( H \) and \( \Gamma \) individually, the optimal execution strategy is determined by the combined impact matrix \( \Theta = \frac{1}{\tau} (H + H^T) - \Gamma \), where \( \tau \) is the time length between consecutive trades. We prove that the minimum expected execution cost strategy is the naive execution strategy, independent of perturbations, when the permanent impact matrix \( \Gamma \) is symmetric and the combined impact matrix \( \Theta \) is positive definite. We provide upper bounds on the size of change in the optimal execution strategy in a general setting. These upper bounds are in terms of the changes in the impact matrices, the eigenvalues of a block tridiagonal matrix defined by \( \Theta \), the risk aversion parameter, and the covariance matrix. These upper bounds indicate that, when the covariance matrix is positive definite, a large risk aversion parameter reduces the sensitivity of the optimal execution strategy. Moreover, when the permanent impact matrix \( \Gamma \) and its perturbation are symmetric, the optimal execution strategy is asymptotically not sensitive to estimation errors when either the minimum eigenvalue of the covariance matrix or the minimum eigenvalue of \( \Theta \) is large. In addition, our computational results confirm that the sensitivity of the optimal execution strategy to the perturbations decreases, when \( \Gamma \) and the perturbed permanent impact matrix are symmetric. Moreover, the change in the efficient frontier increases as the risk aversion parameter decreases for asymmetric perturbations. We consistently observe that imposing short selling constraints can reduce the sensitivity of the optimal execution strategy and the efficient frontier to the perturbations.

Keywords: price impact, execution cost problem, estimation error, sensitivity analysis