Minimizing Makespan of Deteriorating Jobs on a Single Machine

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In this paper, the problem of minimizing makespan of deteriorating jobs on a single machine with unbounded deterioration is considered.The makespan for any schedule is the completion time of the last job. This model was introduced by Browne and Yechiali(1989). There are n independent jobs to be processed on a single machine.Each job j has $p_j(j = 1, 2, \cdots, n)$ as its basic processing time. i.e. when no deterioration has taken place. The jobs, not being processed, begin to deteriorate after a common critical date d and deteriorate no further after common maximal deterioration date $D > d$. If $D < \infty$, then the deterioration is bounded otherwise it is unbounded. Let $w_j; j = 1, 2, \cdots, n$ denote the rates of deterioration of the n jobs. It is assumed here that $\sum_{j=1}^{n} p_j > d$, because otherwise all jobs can begin to be processed before d, and the makespan will be minimized by the SPT sequence with respect to $p_i's$. Let a_j denote the actual processing time for the j^{th} job, where t is the start time of the job, then

$$a_j(t) = \begin{cases} p_j, & \text{if } t \leq d, \\ p_j + w_j(t - d), & \text{if } d \leq t \leq D, \\ p_j + w_j(D - d), & \text{if } t > D. \end{cases} \quad (1)$$

Without loss of generality we assume that d, D and $p_j, w_j, j = 1, 2, \cdots, n$ are all integers. The problem is to find the schedule of jobs which minimizes the makespan, that is, the schedule which minimizes the completion time of the last job of the schedule. The problem is denoted by DET, and has been shown to be NP-hard, when all $p_i's = 1, d = 0$ and $D = \infty$.

Here , we present a polynomial time standard local search algorithm for $d > 0, D = \infty$. The neighborhood for a given schedule is taken as the set of $(n-1)$ adjacent schedules on the convex hull of the $n!$ permutations of $p_i/w_i, i = 1, 2, \cdots, n$ (Theorem(1) Gaiha and Gupta [1977]) . By exploiting the structure of the objective function, we show that by putting certain conditions on the weights, some dominance relations among the set of feasible schedules can be obtained. With the help of these relations we can construct paths decreasing monotonically with respect to the makespan, from any sequence, and bound the number of sequences on such paths. This helps to show that the standard local search algorithm has polynomial time complexity. By relaxing the agreeable rates of deterioration condition we are able to show that the local search algorithm is polynomial time and the locally optimal solution may be not optimal will dominate $O(2^{n-3})$ solutions.

The general problem is NP-hard and this was evident from the fact that the difference formulae were heavily data dependent, thus it was not possible to obtain dominance relations when the rates deterioration are arbitrary. Numerical experiments with 40 odd problems, however suggest that the outcome B of ADJACENT gave solutions within 4% of the optimal solution.